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AN APPROXIMATE METHOD OF SHEAR-LAG ANALYSIS FOR BEAMS LOADED
AT RIGHT ANGLES TO THE PLANE OF SYMMETRY OF THE CROSS SECTION

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RESTRICTED BULLETIN

AN APPROXIMATE METHOD OF SHEAR-LAG ANALYSIS FOR BEAMS LOADED
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SUMMARY

Strain measurements were made on a box beam of triangular cross section and on two beams of D-section under transverse bending. The measured stresses agreed reasonably well with those calculated by a simple adaptation of the substitute-stringer method of shear-lag analysis.

INTRODUCTION

The problem of shear-lag in box beams has been treated by many authors on the assumption that the cross section has at least one plane of symmetry and that the resultant load lies in this plane of symmetry. A few authors have proposed formulas and methods for application to unsymmetrical beams. Among the generally known of these methods, however, there appears to be none which is free from the objection that it disregards some fundamental relation such as the equation of equilibrium of the cross section in the direction of the applied load, the equation of longitudinal equilibrium of the corner flange, or the condition that the shape of the cross section is maintained by bulkheads. In addition, most of these methods suffer from the defect that their application to practical beams of variable cross section and loading is extremely laborious. Confronted with these difficulties, the stress analyst must resort to a simplifying assumption that makes it possible to adapt a theory developed for symmetrical beams. The present paper gives such an adaptation of the substitute-stringer method of analysis to the special case of a box beam with a cross section having one plane of symmetry with the load applied at right angles to this plane. Tests are described that were made to validate this adaptation.

SYMBOLS

A_F	cross-sectional area of effective corner flange, square inches
A_L	cross-sectional area of longitudinal stiffeners, including effective skin, square inches
E	Young's modulus, kips per square inch
G	shear modulus, kips per square inch
K	shear-lag parameter
L	length of beam, inches
b	half-width of actual beam, inches
b'	developed half-width of substitute section, inches
c	camber of substitute section, inches
h	depth of shear web between centroids of flanges, inches
t	thickness of skin, inches
x	distance from tip of beam, inches
y	distance from dividing line of cross section, inches
σ	stress in longitudinal direction, kips per square inch
σ_F	stress in corner flange, kips per square inch
$\sigma_{Mc/I}$	stress in any given stringer calculated by engineering theory of bending, kips per square inch

METHOD OF ANALYSIS

The basic method of shear-lag analysis used in the present paper is that of reference 1, which is based on the use of the so-called substitute single-stringer structure. The method is used in the form described in reference 2, in which the shear-lag effect is calculated as a correction to the standard M_c/I calculation.

For a beam loaded in the plane of symmetry, the substitute single-stringer structure is obtained as indicated in figure 1 by combining all the stringers of the half-section into a single stringer, located at the common centroid. For practical purposes, the half-width b' of the substitute cover may be taken as one-half the actual half-width b .

If an analogous procedure is to be applied to a D-section box such as shown in figure 2, in which the load is normal to the plane of symmetry, a dividing line D-D must be drawn on the cross section as indicated in the figure. As long as no acceptable theory of unsymmetrical sections is available, some assumption must be made concerning the location of the dividing line and the assumption must then be validated by tests. As a trial, it was assumed that the dividing line could be drawn halfway between the shear web and the nose, and it will be shown that this assumption gave acceptable results for the maximum longitudinal stresses in three test specimens. No attempt was made to calculate the stresses in the nose portions of the beams, because the approximate method employed herein is clearly inapplicable in the region of the nose.

For a substitute section such as shown in figure 2, the shear-lag parameter K is defined by

$$K^2 = \frac{Gt}{Eb'} \left(\frac{1 + \frac{2c}{h}}{A_F} + \frac{1}{A_L} \right) \quad (1)$$

and the stress in the flange is given by

$$\begin{aligned} \sigma_F &= \sigma_{Mc}/I + \Delta\sigma_F \\ &= \sigma_{Mc}/I \left[1 + \frac{A_L}{A_F} \left(1 + \frac{2c}{h} \right)^2 \frac{\sinh Kx}{Kx \cosh KL} \right] \end{aligned} \quad (2)$$

The signs in formulas (1) and (2) apply to beams with positive camber as shown in figure 1; in the D-sections considered herein, the camber c is negative, the depth of the section at the shear web being greater than the depth at the dividing line.

In a stringer located at a distance y from the dividing line, the stress is obtained by adding to the stress $\sigma_{Mc/I}$ calculated by the ordinary bending theory, a correction that is given by the formula

$$\Delta\sigma = \Delta\sigma_F - \frac{4}{3} \Delta\sigma_F \left[1 + \frac{A_F}{A_L \left(1 + \frac{2c}{h} \right)} \right] \left[1 - \left(\frac{y}{b} \right)^3 \right] \quad (3)$$

Formula (3) is an approximation (reference 2) to another formula which is derived on the assumption that the camber is negligible. If the camber is appreciable, the formula should be used only for the region near the flange ($y/b \rightarrow 1$). Fortunately, only this region is of importance in design.

TEST SPECIMENS AND TEST PROCEDURE

The dimensions of two of the three beams are shown in figure 3. The third beam was made by cutting the D-section shown in figure 3(b) along the line B-B. The material was 24S-T aluminum alloy. The value of Young's modulus was taken as 10,500 kips per square inch.

Each beam was supported at the center. Two equal loads were applied, one to each half of the beam, to produce the bending moments. The structure as well as the loading being symmetrical about the root, each half of the beam could be considered as a cantilever with a perfectly fixed root. On the beam with triangular cross section, each load was applied close to the tip of the beam. On the other beams, the loads were applied at the middle of the semispan in order to increase the shear-lag effect. The loads were applied to each beam at the experimentally determined shear center.

Strain measurements were made with Tuckerman optical strain gages with a gage length of 2 inches. The load was applied in three equal increments. Check runs were made if the load-stress diagram indicated a zero error of more than 0.2 kip per square inch.

CALCULATED AND EXPERIMENTAL RESULTS

As previously stated, the shear-lag calculations were based on substitute single-stringer structures representing the part of the box between the shear web and a dividing line drawn halfway between the shear web and the nose. The substitute structures are shown in figure 4. The substitute structure for the D-section with overhang is an I-section, which must be split in two before the formulas can be applied. Before the I-section was split, it was made symmetrical by averaging the values of A_L , of b' , and of c shown in figure 4. This simplification was considered justifiable because the section was approximately symmetrical and because the entire calculation is only a first approximation.

The area A_T is made up of four items: the area of the flanged portion of the shear web, an area equal to one-sixth of the web area that accounts for participation of the web in the bending action, the portion of the skin from the line of flange rivets halfway to the adjacent stringer (or stringers, in the case of the D-section with overhang), and the remaining strip in contact with the flanged part of the shear web. The area A_L is made up of two items: the area of the longitudinal stiffeners included between the shear web and the dividing line, and the area of the skin lying between the shear web and the dividing line excepting the portion of skin considered as part of A_T . The skin was assumed to be fully effective because the measurements were taken on the tension side of the beam.

In reference 1, it was stated that the strip of skin adjacent to the flange should be considered as part of A_L rather than A_T , as it is considered in the present paper. The procedure of reference 1 is recommended for design purposes, but the modified procedure used herein is believed to be better when it is intended to compare calculated stresses with stresses measured in the skin on top of the flange, instead of with stresses in the flange itself. In the test beams, the maximum flange stresses calculated by the two procedures differ by roughly 10 percent. In most practical beams, the difference would be much smaller because a distinct corner flange would be provided.

Formula (2) was derived for beams loaded at the tip. A study of the general theory (reference 1) shows, however, that the formula may be used for calculating the stresses near the root caused by a load applied anywhere within the span if the distance from the root to the load is substituted for L in the formula. The results obtained in this manner are sufficiently accurate provided the value of the parameter KL is not too small - say, $KL > 3$. This condition was fulfilled for the D-section beams.

The calculated and the experimental results are shown in figures 5 to 7. The chordwise distribution of stresses calculated by the shear-lag theory is shown up to the dividing line ($y = 0$), although the calculation should not be considered as valid except near the flange, as mentioned in Method of Analysis.

Because the dividing line was drawn according to an arbitrary assumption, the agreement between calculated and test results is satisfactory for the stresses in the flange and in the stringer adjacent to the flange, which are the stresses dominating the design.

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2. Kuhn, Paul: A Procedure for the Shear-Lag Analysis of Box Beams. NACA A.R.R., Jan. 1943.

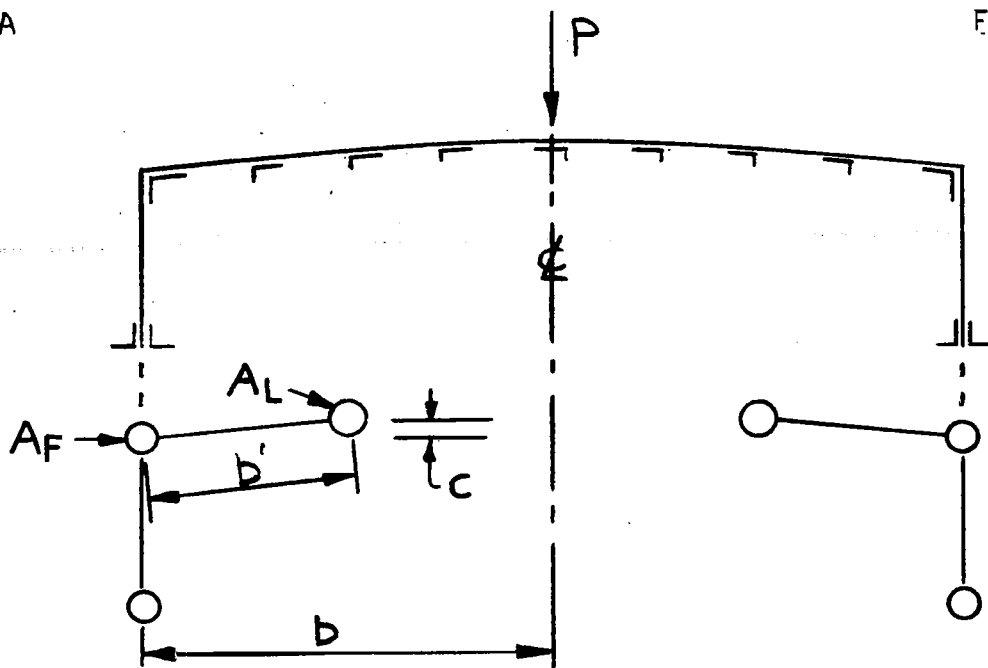


Figure 1.- Actual and substitute section of beam loaded in the plane of symmetry.

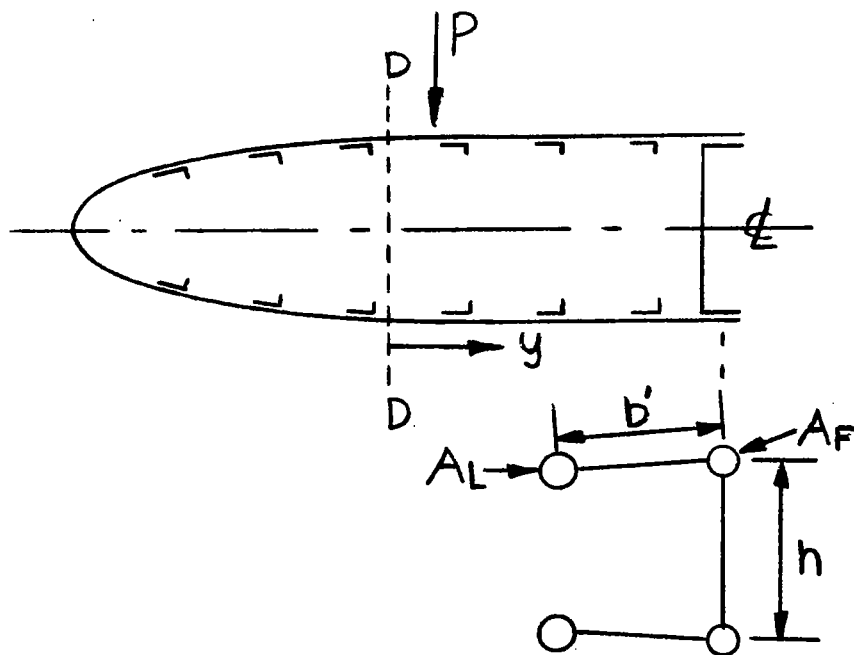
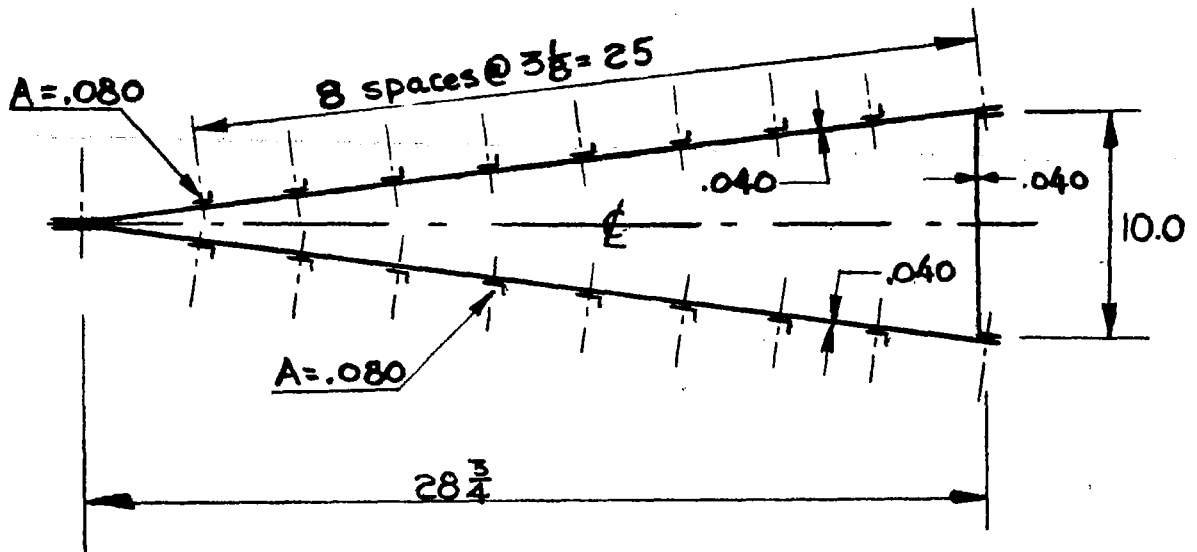
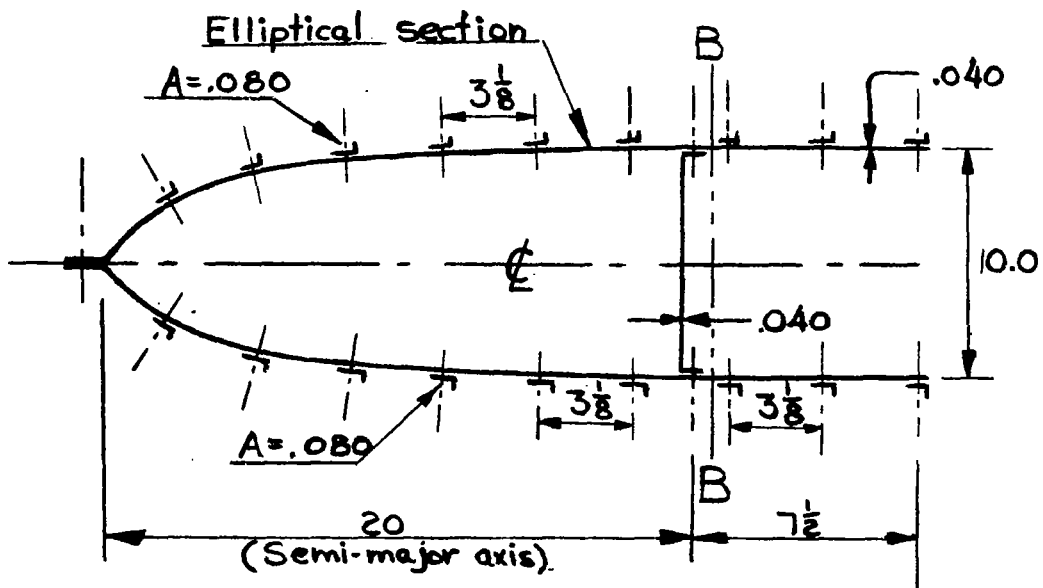


Figure 2.- Actual and substitute section of beam loaded at right angles to plane of symmetry.



(a) Triangular box.



(b) D-section with overhang.

Figure 3.- Cross sections of test beams.

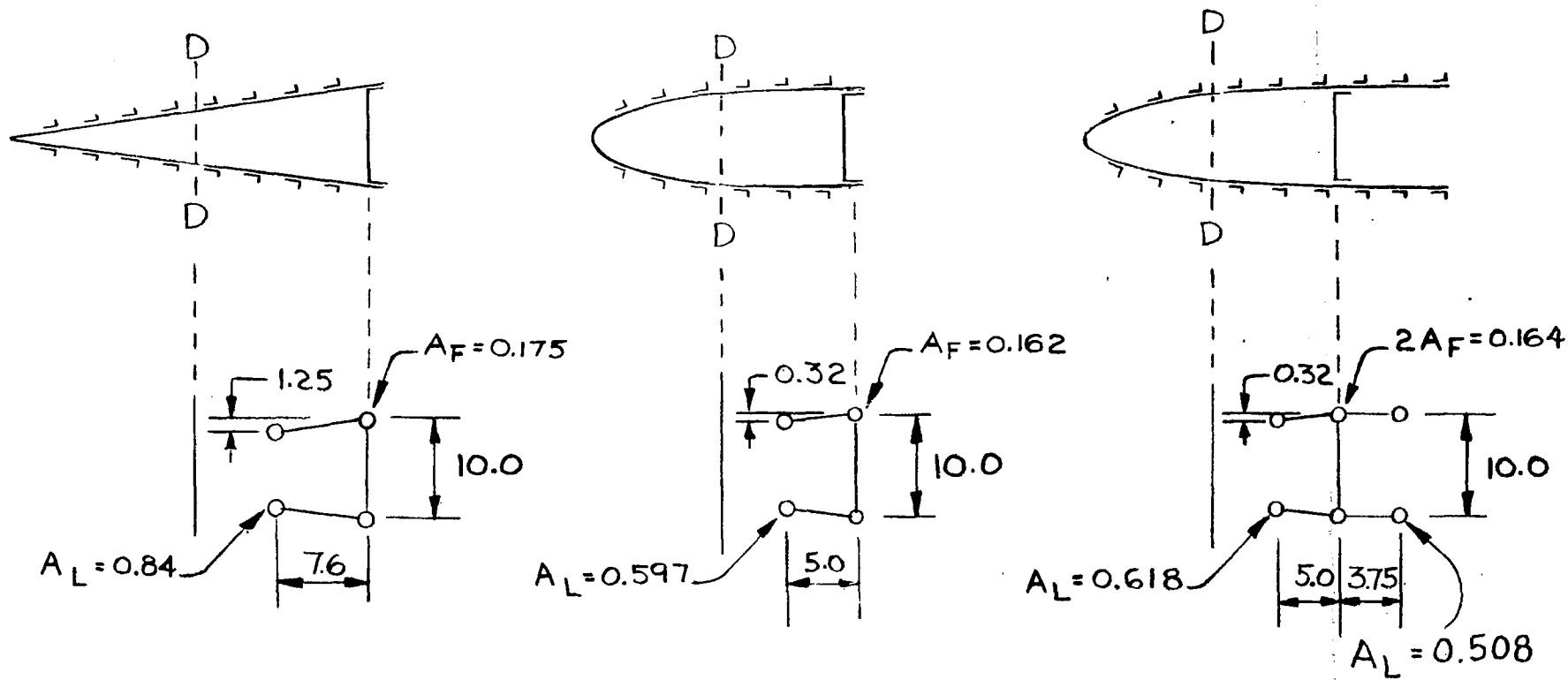
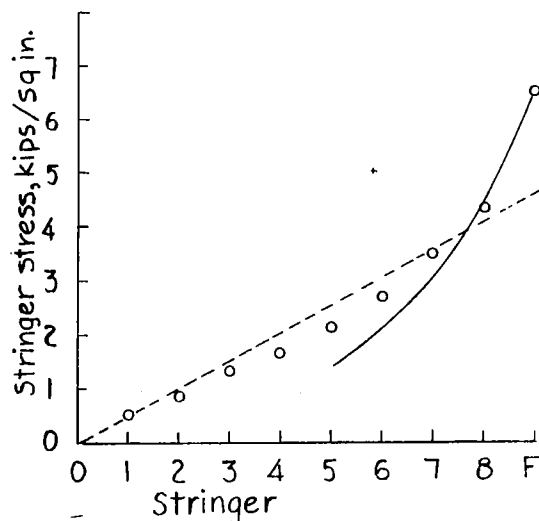
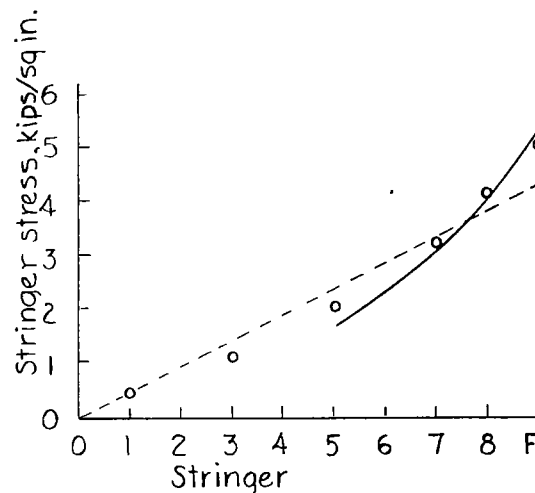


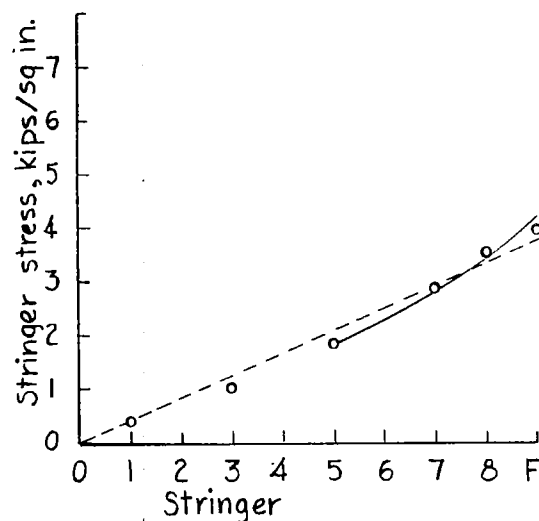
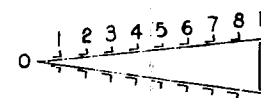
Figure 4.- Actual and substitute sections of test beams.



Chordwise stress distribution, 25 inches from root.



Chordwise stress distribution, 7.25 inches from root.



Chordwise stress distribution, 14 inches from root.

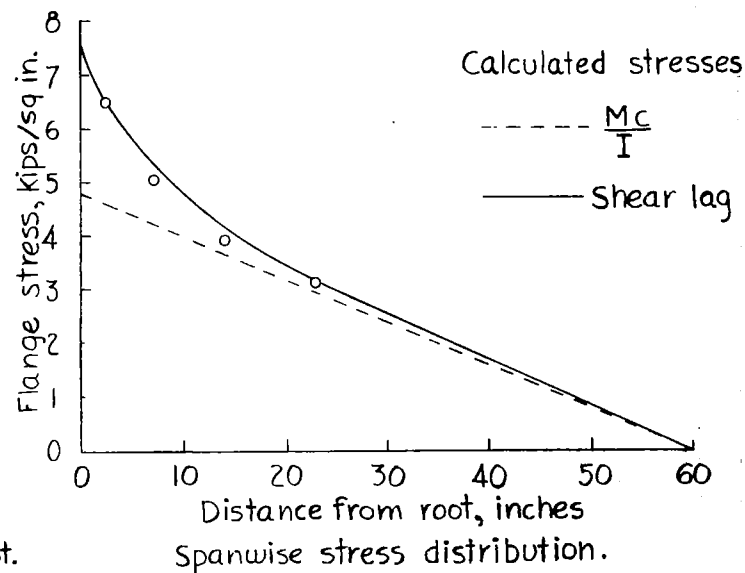
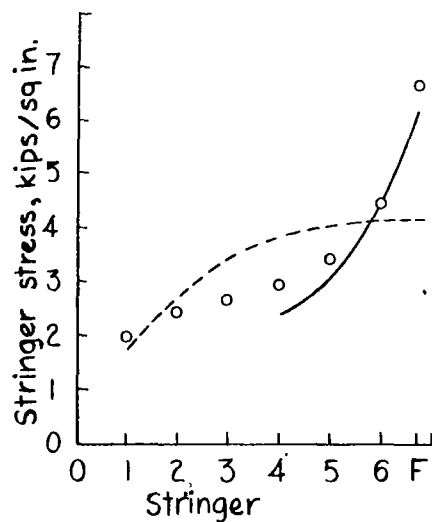
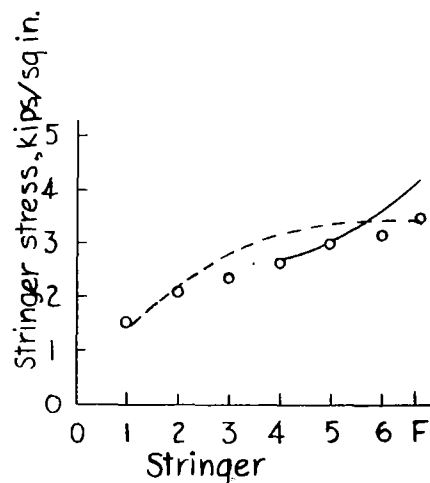


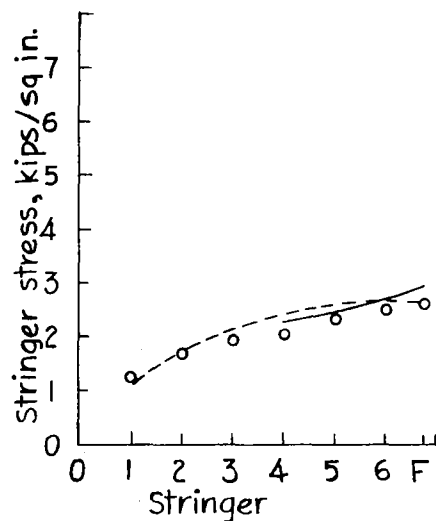
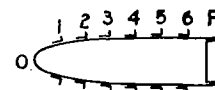
Figure 5.- Experimental and calculated stresses in triangular box.



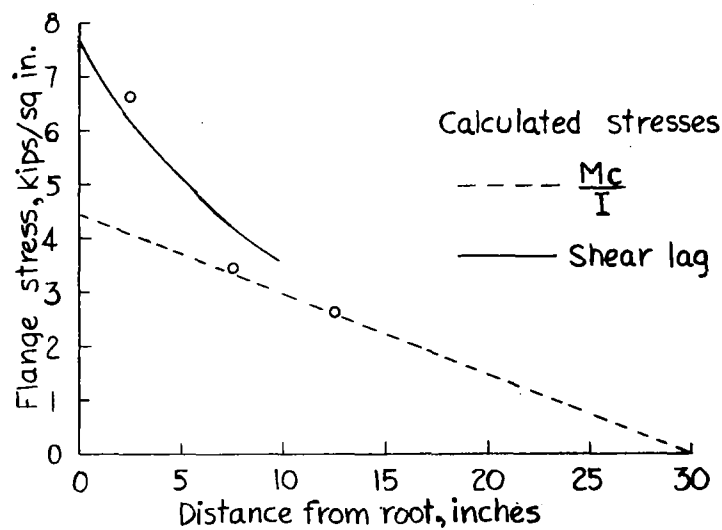
Chordwise stress distribution, 2.5 inches from root.



Chordwise stress distribution, 7.5 inches from root.

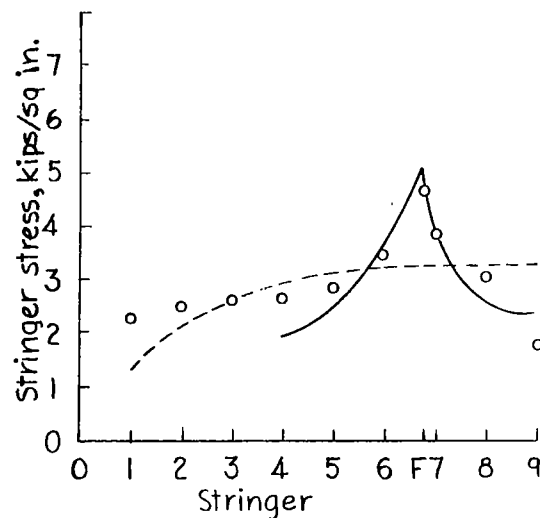


Chordwise stress distribution, 12.5 inches from root.

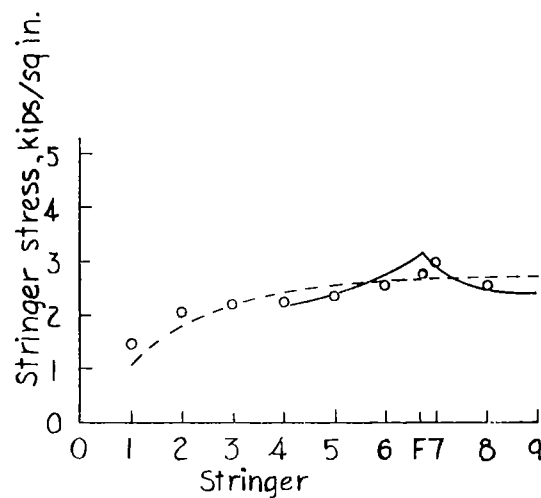


Spanwise stress distribution.

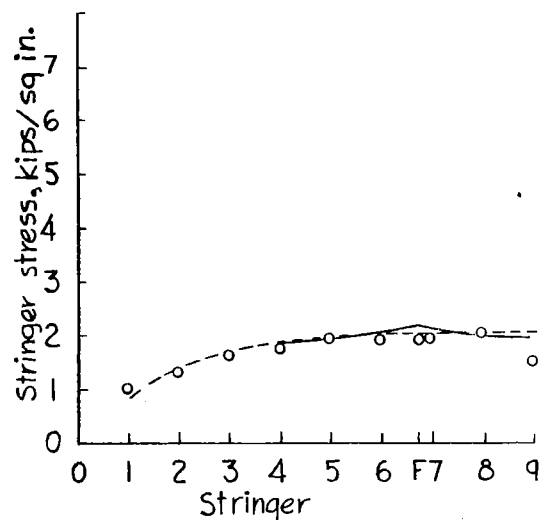
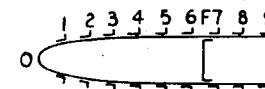
Figure 6.- Experimental and calculated stresses in D-section box.



Chordwise stress distribution, 2.5 inches from root.



Chordwise stress distribution, 7.5 inches from root.



Chordwise stress distribution, 12.5 inches from root.

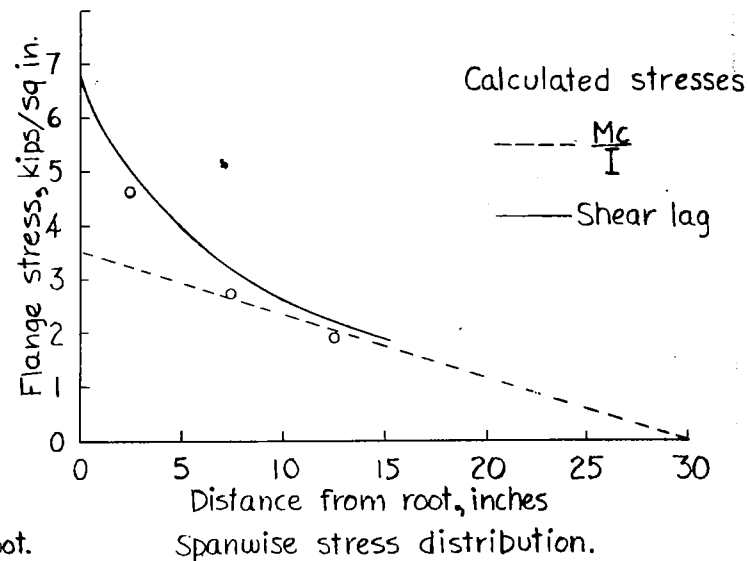


Figure 7.- Experimental and calculated stresses in D-section with overhang.

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